

to be an astringent mineral earth; whereas charcoal easily takes fire, burns freely without smoke, and continues burning, till it consumes to an ash; which consists of an alkaline salt, and a pure earth, fit for making cuppels; and, by these marks, is sufficiently distinguished from all mineral substances.

Grosvenor-Street, Jan. 8, 1761.

Jer. Milles.

LXXXVI. *De Aberratione Luminis, in Superficiebus et Lentibus Sphæricis refractorum* *.

§ 1.

Read April 2,
1761.

SI radius luminis AG, incidens in superficiem quamcunque refringentem LE, inflectatur secundum rectam GK, et quævis recta linea AK occurrat radio incidenti AG in A, refracto GK in K; et rectæ GD, normali ad superficiem refringentem LE, in D; erit rectangulum GK × DA ad rectangulum GA × DK, ut finus anguli incidentiæ DGA ad finum anguli refractionis DGK. Est enim DA ad GA, ut fin. DGA ad fin. ADG, et GK : DK :: fin. GDK : fin. DGK; quare, ob

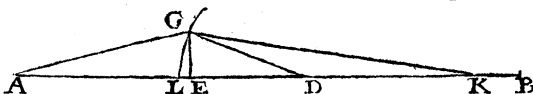
* This Paper, though sent to England in the summer of the year 1760, was, by accident, prevented from being read to the Royal Society, till the 2d of April following.

fin.

fin. $ADG = \text{fin. } KDG$, erit componendo $DA \times GK : GA \times DK :: \text{fin. } DGA : \text{fin. } DGK$. Quæ ratio, si ponatur ut i ad r , erit $i \cdot GA \times DK = r \cdot GK \times DA$.

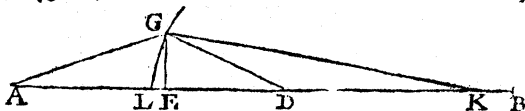
§ 2.

Incidat radius luminis AG in superficiem sphaericam LG ,
cujus centrum est D ,
et refringatur secundum rectum GK , quæritur concursus K radii refracti GK cum axe sphaeræ ALD , posito arcu LG satis parvo.



A puncto incidentiæ G cadat GE normalis ad axem AD , ducaturque radius sphaeræ GD . His factis est per elementa, $AGq = ADq + DGq - 2AD \times DE = ADq + DLq - 2AD \times \overline{DL - LE} = \overline{AD - DL}q + 2AD \times LE = \overline{AL}q + 2AD \times LE$, adeoque $AG = \sqrt{\overline{AL}q + 2AD \times LE} = (\text{ob } LE \text{ satis parvam}) \overline{AL} + \frac{AD \times LE}{AL}$ quamproxime. Similiter $KGq = KDq + DGq + 2KD \times DE = KDq + DLq + 2KD \times \overline{DL - LE} = \overline{KD + DL}q - 2DK \times LE = \overline{KL}q - 2KD \times LE$, adeoque $KG = \sqrt{\overline{KL}q - 2KD \times LE} = \overline{KL} - \frac{KD \times LE}{KL}$ quamproxime.

Jam vero est (§ 1.) $i \cdot GA \times DK = r \cdot GK \times DA$, quare substitutis valoribus modo in-



ventis ipsarum GA et GK , habetur $i \cdot KD \times (\overline{AL} + \frac{AD \times LE}{AL}) = r \cdot AD \times (\overline{KL} - \frac{KD \times LE}{KL})$, et

trans-

transponendo $r. AD \times KL - i. KD \times AL = AD \times KD \times LE \times \left(\frac{i}{AL} + \frac{r}{KL} \right)$.

Ponatur jam $LA = A$, $LK = K$, radius sphaerae $DL = a$, adeoque $AD = A + a$, et $DK = K - a$. His in novissima æquatione scriptis, erit $rK \times \overline{A + a} - iA \times \overline{K - a} = \overline{A + a} \times \overline{K - a} \times LE \times \left(\frac{i}{A} + \frac{r}{K} \right)$, unde transponendo et dividendo, $K = \frac{iAa}{i - r. A - ra}$

$- \frac{\overline{A + a} \times \overline{K - a} \times LE}{i - r. A - ra} \times \left(\frac{i}{A} + \frac{r}{K} \right)$.

Jam si radius incidens AG fuerit axi AD vicinissimus, evanescet LE , adeoque et terminus $\frac{\overline{A + a} \times \overline{K - a} \times LE}{i - r. A - ra} \times \left(\frac{i}{A} + \frac{r}{K} \right)$. Quare in hoc casu

$K = \frac{iAa}{i - r. A - ra}$, id est, distantia foci geometrici

ipfi A conjugati à vertice L , erit $\frac{iAa}{i - r. A - ra}$, unde

aberratio radii refracti GK ab hoc foco erit $\frac{\overline{A + a} \times \overline{K - a} \times LE}{i - r. A - ra} \times \left(\frac{i}{A} + \frac{r}{K} \right)$, fumendo à foco

contra directionem cursus radiorum. Quoniam vero aberratio illa semper est valde parva, erit $\frac{iAa}{i - r. A - ra}$

valor prope verus distantiae LK five K , adeoque in expresseione aberrationis sine sensibili errore pro K usurpari potest. Sit itaque distantia LB foci geometrici B à vertice superficie L , five $\frac{iAa}{i - r. A - ra} = B$,

eritque

eritque K five LB = $B - \frac{A+a \cdot B-a \cdot LE}{i-r \cdot A-ra} \times \left(\frac{i}{A} + \frac{r}{K} \right)$
quamproxime.

Dimidia latitudo superficiei refringentis, five distantia puncti incidentiæ G ab axe, id est LG, dicatur L, erit quamproxime $LE = \frac{L^2}{2a}$: quare in formula modo inventa pro LE scribendo $\frac{L^2}{2a}$, et ulterius concinnando expressionem aberrationis ope æquationis $B = \frac{i A a}{i-r \cdot A-ra}$, habetur formula, qua in sequentibus præcipue utemur: $LK = B - \frac{r \cdot i-r \cdot B^2 L^2}{2 i^3} \times \left(\frac{1}{a} + \frac{1}{A} \right)^2 \times \left(\frac{r}{a} + \frac{i+r}{A} \right)$.

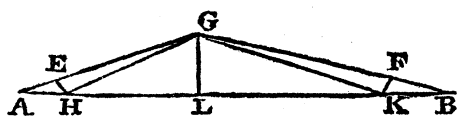
§ 3.

In formulis jam inventis signa symbolorum illi tantum casui sunt accommodata, quem figura exprimit, ubi radius luminis ab axe divergens ponitur incidere in superficiem convexam, et post refractionem ad axem convergere. Accommodantur vero ad reliquos problematis casus, mutando signum radii sphaeræ a , si radius luminis incidat in superficiem concavam, et signum ipsius A , si radius incidentis convergat ad axem. Hoc facto, si valor distantiae LB five B prodit positivus, sumenda est distantia illa à vertice L secundum directionem cursus radiorum; si negativus, contra eandem directionem. Aberratio vero radii à foco B, five BK per formulas computata, si fuerit positiva, sumenda est à foco B contra directionem cursus radiorum; si negativa, secundum hanc directionem. Si radii incidentes fuerint axi pa-

ralleli, facienda est A infinita, et si B prodit infinita, radii refracti erunt axi paralleli, abstrahendo ab illorum aberratione.

§ 4.

Si duo radii luminis AG , HG , in angulo quam



minimo AGH inter se inclinati incident fere perpendiculariter in

idem punctum G superficiei cujuscunque refringentis LG , et refringantur, prior in GK , posterior vero in GB , occurrentes lineæ rectæ cuicunque in punctis A , H , K , B ; dico esse BK ad AH ut est $r \cdot LBq$ ad $i \cdot LAq$, posita i ad r ut sinus incidentiæ ad sinum refractionis.

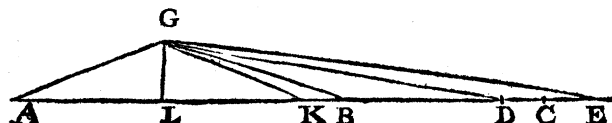
Quoniam enim anguli valde parvi sunt finibus suis quamproxime proportionales, erunt in parvis refractionibus anguli incidentiæ et refractionis eorumque adeo differentiæ finibus incidentiæ et refractionis proportionales quamproxime. Est itaque angulus AGH , utpote differentia angulorum incidentiæ radiorum AG , HG , ad angulum BGK , sive differentiam angulorum refractionis radiorum GK , GB , ut i ad r . Quare centro G inter hos angulos descriptis arcubus HE , KF , erit HD ad KF , ut $i \cdot GH$ ad $r \cdot GK$;

sed est $AH = \frac{GH \cdot HE}{GL}$, et $BK = \frac{GK \cdot KF}{GL}$, quare

pro HE et KF , scribendo earum proportionales $i \times GH$ et $r \times GK$, habetur AH ad BK ut $i \times GHq$ ad $r \times GKq$, quamproxime, id est, ut $i \times LAq$ ad $r \times LBq$.

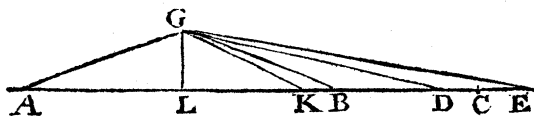
§ 5.

Data forma lentis refringentis, et data in ejus axe distantia foci radiorum incidentium, invenire punctum concursus radii cujusvis refracti cum axe.



Sit locus lentis in L , ejus axis $ALBC$, dimidia latitudo LG , five distantia incidentiæ ab axe, $= L$; radius convexitatis superficiæ primæ, in quam radii incident, $= a$; radius convexitatis superficiæ secundæ, è qua emergunt, $= b$, radius luminis quivis incidens AG , refractus GK . In axe lentis ABC sit distantia foci radiorum incidentium, five $LA = A$, distantia foci geometrici ipsi A conjugati, five $LB = B$; distantia foci geometrici C superficiæ primæ ipsi A conjugati, five $LC = C$, et evidens est, esse etiam idem punctum C focus geometricum superficiæ secundæ conjugatum ipsi B . Sit tandem distantia puncti concursus K radii refracti GK cum axe, five $LK = K$.

Radius AG in prima superficie refractus tendat versus punctum D , et concipiatur à puncto B , quod est focus lentis ipsi A conjugatis, radius BG incidere



in secundam superficiem, et in ea ita refringi, ut divergat à puncto axeos E . Et dabuntur per § 2. aberrationes CE et CD , adeoque earum summa DE . Sed per § 4. est DE ad BK ut $r \cdot LCq$ ad $i \cdot LBq$,

quare $BK = \frac{i \cdot LBq}{r \cdot LCq} \times DE = \frac{iB^2}{rC^2} \times DE$. Jam per

§ 2. est $CD = \frac{r \cdot \overline{i-r} \cdot C^2 L^2}{2i^3} \times \left(\frac{1}{a} + \frac{1}{A}\right) \times \left(\frac{r}{a} + \frac{i+r}{A}\right)$

et $CE = \frac{r \cdot \overline{i-r} \cdot C^2 L^2}{2i^3} \times \left(\frac{1}{b} + \frac{1}{B}\right) \times \left(\frac{r}{b} + \frac{i+r}{B}\right)$,

quarum summa DE , ducta in $\frac{i \cdot B^2}{r \cdot C^2}$, dat aberrationem

quæsitam $BK = \frac{iB^2}{rC^2} \times DE = \frac{\overline{i-r} \cdot B^2 L^2}{2i^2} \left[\left(\frac{1}{a} + \frac{1}{A}\right)^2 \times \left(\frac{r}{a} + \frac{i+r}{A}\right) + \left(\frac{1}{b} + \frac{1}{B}\right)^2 \times \left(\frac{r}{b} + \frac{i+r}{B}\right) \right]$, quæ

post evolutionem terminorum subtracta de LB , five B , relinquit distantiam puncti concursus $LK =$

$$B - \frac{\overline{i-r} \cdot B^2 L^2}{2i^2} \times \left\{ \frac{r}{a^3} + \frac{i+3r}{Aa^2} + \frac{2i+3r}{A^2a} + \frac{i+r}{A^3} \right. \\ \left. + \frac{r}{b^3} + \frac{i+3r}{Bb^2} + \frac{2i+3r}{B^2b} + \frac{i+r}{B^3} \right\}.$$

Porro quoniam punctum C est focus ipsi A conjugatus respectu faciei primæ, erit $LC = \frac{iAa}{\overline{i-r} \cdot A - ra}$;

et quoniam idem punctum C est focus ipsi B conjugatus respectu superficiei secundæ, erit $LC =$

$$- \frac{iBb}{\overline{i-r} \cdot B - rb}, \text{ per § 2. Hinc } \frac{iAa}{\overline{i-r} \cdot A - ra} =$$

$$- \frac{iBb}{\overline{i-r} \cdot B - rb}, \text{ adeoque } B = \frac{rAab}{\overline{i-r} \cdot A \cdot a + b - rab},$$

five $\frac{1}{A} + \frac{1}{B} = \frac{i-r}{r} \times \frac{1}{a} + \frac{1}{b}$. Dato sic valore ipsius

B , cognoscitur distantia concursus radii refracti GK cum axe five LK , ejusque aberratio à foco B five BK , quæ ob signum — sumenda est à foco B contra directionem

directionem curfus radorum, si valorem habet pō-
sitivum; secundum vero, si negativum.

Accommodantur vero hæ formulæ ad reliquos pro-
blematis casus eadem ratione ac dictum est § 3. de
aberratione superficiæ simplicis.

§ 6.

Expressio aberrationis jam inventa, ad formam re-
ducitur usui magis accommodatam, conjungendo ter-
minos, in quibus a et b easdem habent dimensiones,
et reducendo singulas partes ope æquationis $\frac{i-r}{r} \times \frac{1}{a} + \frac{1}{b}$
 $= \frac{1}{A} + \frac{1}{B} = \frac{1}{P}$, ubi P est distantia foci principalis.

$$\begin{aligned} \text{Hac ratione calculum subducendo habetur: } & \frac{1}{a^3} + \frac{1}{b^3} \\ &= \frac{r}{i-r.P} \times \left(\frac{r^2}{i-r^2.P^2} - \frac{3r}{i-r.P.a+b} \right); \frac{1}{Aa^2} + \frac{1}{Bb^2} \\ &= \frac{r}{i-r.P} \times \left(\frac{1}{Aa} + \frac{1}{Bb} - \frac{1}{P.a+b} \right); \frac{1}{A^2a} + \frac{1}{B^2b} = \\ &= \frac{r}{i-r.P} \times \left(\frac{i-r}{r} \times \frac{1}{Aa} + \frac{1}{Bb} - \frac{1}{P.A+B} \right); \frac{1}{A^3} + \frac{1}{B^3} \\ &= \frac{r}{i-r.P} \times \left(\frac{i-r}{rP^2} - \frac{3.i-r}{rP.A+B} \right). \end{aligned}$$

Singulæ harum partium ductæ in suos respective
coëfficientes, $r, i+3r, 2i+r, i+r$, conjungantur
cum signis propriis, et habebitur: $LK = LB - BK =$
 $B - \frac{B^2L^2}{2iP^2} \left(\frac{i^3-2i^2r+2r^3}{i-r^2.P} - \frac{3i+2r}{A+B} + \frac{2.i+r}{A+B} \times \frac{A+B}{b} + \frac{A}{b} \right.$
 $\left. - \frac{r.i+2r}{i-r.a+b} \right)$, vel etiam $= B - \frac{B^2L^2}{2iP^2} \times \left(\frac{i^3}{i-r^2.P} \right.$
 $\left. - \frac{3i+2r}{A+B} - \frac{2.i+r}{A+B} \times \frac{A}{a} + \frac{B}{b} - \frac{r.i+2r}{i-r.a+b} \right)$.

§ 7.

Invenire lentem, quæ radios luminis à data distantia advenientis in alia data distantia colligat cum minima aberratione, ipsamque aberrationem minimam.

Retentis omnibus symbolis ut supra, sumantur differentialia aberrationis modo inventæ, $\frac{B^2 L^2}{2i \cdot P^2} \times \left(\frac{i^3}{(i-r)^2 \cdot P} \right.$

$$\left. - \frac{3i+2r}{A+B} - \frac{2 \cdot \overline{i+r}}{A+B} \times \frac{\overline{A}}{a} + \frac{\overline{B}}{b} - \frac{r \cdot \overline{i+2r}}{i-r \cdot \overline{a+b}} \right),$$

positis a et b variabilibus, et habebitur $\frac{2 \cdot \overline{i+r}}{A+B}$

$$\times \frac{\overline{A da}}{a^2} \times \frac{\overline{B db}}{b^2} + \frac{r \cdot \overline{i+2r}}{i-r} \times \frac{\overline{da+db}}{a+b} = 0; \text{ sed ob}$$

$\frac{1}{a} + \frac{1}{b} = \frac{r}{i-r \cdot P}$, est $\frac{da}{a^2} + \frac{db}{b^2} = 0$; quare tollendo

per comparationem harum æquationum differentialia,

$$\text{invenitur: } \frac{b-a}{b+a} = \frac{2 \cdot \overline{i+r} \cdot \overline{i-r}}{r \cdot \overline{i+2r}} \times \frac{A-B}{A+B}, \text{ adeoque}$$

$$\frac{a}{b} = \frac{4r^2 + ri - 2i^2 \cdot A + i \cdot 2 \cdot \overline{i+r} \cdot B}{4r^2 + ri - 2i^2 \cdot B + i \cdot 2 \cdot \overline{i+r} \cdot A}; \text{ unde per æqua-}$$

tionem $\frac{1}{a} + \frac{1}{b} = \frac{r}{i-r} \times \frac{1}{A} + \frac{1}{B}$, colliguntur radii facierum

$$\text{lentis quæsitæ, scil. } a = \frac{2 \cdot \overline{i-r} \cdot \overline{i+2r} \cdot AB}{i \cdot 2 \cdot \overline{i+r} \cdot A + 4r^2 + ri - 2i^2 \cdot B},$$

$$\text{et } b = \frac{2 \cdot \overline{i-r} \cdot \overline{i+2r} \cdot AB}{i \cdot 2 \cdot \overline{i+r} \cdot B + 4r^2 + ri - 2i^2 \cdot A}; \text{ five } \frac{1}{a} =$$

$$\frac{i \cdot 2 \cdot \overline{i+r}}{2 \cdot \overline{i-r} \cdot \overline{i+2r}} \times \frac{1}{B} + \frac{4r^2 + ri - 2i^2}{2 \cdot \overline{i-r} \cdot \overline{i+2r}} \times \frac{1}{A}; \text{ et } \frac{1}{b} =$$

$$\frac{i \cdot 2 \cdot \overline{i+r}}{2 \cdot \overline{i-r} \cdot \overline{i+2r}} \times \frac{1}{A} + \frac{4r^2 + ri - 2i^2}{2 \cdot \overline{i-r} \cdot \overline{i+2r}} \times \frac{1}{B}.$$

Hic

Hic valores radorum a et b substituti in formula aberrationis generali § 6. dant ipsam aberrationem

$$\text{minimam} = \frac{i \cdot B^2 L^2}{2 \cdot i + 2r \cdot P^2} \times \left(\frac{P}{A + B} + \frac{r \cdot \overline{4i - r}}{4 \cdot \overline{i - r^2}} \right).$$

§ 8.

Postquam in § præcedente formam lentis invenimus, quæ in datis circumstantiis aberrationem minimam producat, et simul ipsam ejus aberrationem; rationi consentaneum est, investigationem aberrationis lentium ab hac forma discendentium ita aggredi, ut simul appareat relatio formarum harum lentium ad illius formam, et aberrationis harum ad illius aberrationem. Solent enim conclusiones prodire concinniores, si ad casum inter reliquos maxime singularem et unicum referantur, qualis hic est casus aberrationis minimæ.

Scribatur brevitatis causa b pro $\frac{i \cdot \overline{2i + r}}{2 \cdot \overline{i - r} \cdot \overline{i + 2r}}$, et K pro $\frac{4r^2 + ri - 2i^2}{2 \cdot \overline{i - r} \cdot \overline{i + 2r}}$, et erit pro determinandis radiis a et b lentis aberrationis minimæ, $\frac{r}{a} = \frac{b}{B} + \frac{K}{A}$, et $\frac{r}{b} = \frac{b}{A} + \frac{K}{B}$, per § 7. Ponatur jam generaliter pro radiis a et b facierum cujuscvis alius lentis, $\frac{r}{a} = \frac{b}{B} + \frac{K}{A} + \frac{x}{P}$, et $\frac{r}{b} = \frac{b}{A} + \frac{K}{B} - \frac{x}{P}$; quos valores ipsarum $\frac{r}{a}$ et $\frac{r}{b}$ tales assumo, ut earum summa $\frac{r}{a} + \frac{r}{b}$ fit $= \overline{\frac{b}{A} + K} \times \overline{\frac{r}{A} + \frac{r}{B}}$, sive $\frac{r}{i - r} \times \overline{\frac{r}{A} + \frac{r}{B}}$, quemadmodum

dum oportet. Substituantur hi valores ipsarum $\frac{1}{a}$ et $\frac{1}{b}$ in formula aberrationis generali, $\frac{B^2 L^2}{2 i P^2} \times \left(\frac{i^3}{P} - \frac{3i + 2r}{A + B} - \frac{2 \cdot \overline{i + r}}{A + B} \times \frac{\overline{A}}{a} + \frac{\overline{B}}{b} - \frac{r \cdot \overline{i + 2r}}{i - r \cdot \overline{a + b}} \right)$ § 6. et peracto calculo invenietur $\frac{i B^2 L^2}{2 \cdot \overline{i + 2r} \cdot P^3} \left(\frac{P}{A + B} + \frac{\overline{4i - r \cdot r}}{4 \cdot \overline{i - r}^2} \cdot \frac{\overline{i + 2r}^2}{i^2} \right) N^2$) pro aberratione lentis, cujus facies luminis obversa radium habeat a , et facies à lumine averfa radium habeat b , existentibus $\frac{1}{a} = \frac{b}{B} + \frac{K}{A} + \frac{x}{P}$, et $\frac{1}{b} = \frac{b}{A} + \frac{K}{B} - \frac{x}{P}$; P distantia foci principalis, A distantia foci radiorum incidentium, et B refractorum.

Commodum itaque hic accidit, ut numerus x , qui relationem formæ lentis ad lentem aberrationis minimæ designat, idem quoque relationem aberrationis ejus ad aberrationem minimam ratione non minus simplici exhibeat.

§ 9.

Si in formulis § præced. exhibitis, pro radiis facierum lentis ejusque aberratione, statuatur $x = 0$, radit lens ad eam formam, quæ aberrationem dat minimam, et aberratio ejus ad aberrationem minimam. Si vero non fuerit $x = 0$, apparet ex iisdem formulis, cuivis aberrationis quantitati, quæ major fit aberratione minima, duas diversas respondere lentis formas, quarum utraque definitur æquationibus $\frac{1}{a} = \frac{b}{B} + \frac{K}{A} + \frac{x}{P}$, et $\frac{1}{a} = \frac{b}{A} + \frac{K}{B} - \frac{x}{P}$, ubi x pro altera

altera positive, altera negative accipitur. Nam in formula aberrationis $\frac{i B^2 L^2}{2 \cdot \overline{i} + 2r \cdot P^2} \times \left(\frac{P}{A+B} + \frac{\overline{4i-r} \cdot r}{4 \cdot \overline{i-r}^2} + \frac{\overline{i+2r}^2}{i^2} x^2 \right)$ non comparet nisi quadratum ipsius x , quod idem manet, five radix x sumatur positive, five negative.

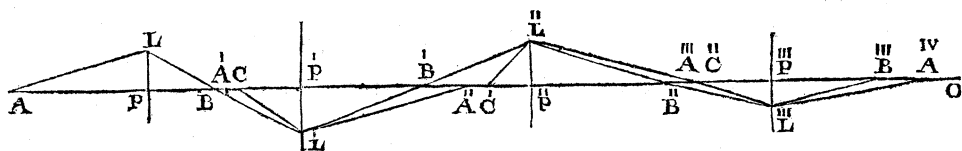
Quoniam aberratio quævis $\frac{i B^2 L^2}{2 \cdot \overline{i} + 2r \cdot P^2} \times \left(\frac{P}{A+B} + \frac{\overline{4i-r} \cdot r}{4 \cdot \overline{i-r}^2} + \frac{\overline{i+2r}^2}{i^2} x^2 \right)$ est ad aberrationem minimam $\frac{i B^2 L^2}{2 \cdot \overline{i} + 2r \cdot P^2} \times \left(\frac{P}{A+B} + \frac{\overline{4i-r} \cdot r}{4 \cdot \overline{i-r}^2} \right)$ in ratione majoris inæquabilitatis, statuatur illa ratio $1 + m^2$ ad 1 ; assumendo m pro numero quovis, et invenitur $x = \pm \frac{mi}{i+2r} \sqrt{\left(\frac{P}{A+B} + \frac{\overline{4i-r} \cdot r}{4 \cdot \overline{i-r}^2} \right)}$. Et sic ex data ratione aberrationis alicujus lentis ad aberrationem minimam, datur illi correspondens numerus x , adæque et radii facierum ejus, per superiora.

Numerus hic x , quoniam simul et formam et aberrationem suæ lentis indicat, dicatur *index* lentis, et scribatur brevitatis causa f pro $\frac{\overline{4i-r} \cdot r}{4 \cdot \overline{i-r}^2}$, et g pro $\frac{i+2r}{i}$, ut aberratio sit $\frac{B^2 L^2}{2g \cdot P^2} \times \left(\frac{P}{A+B} + f + g^2 x^2 \right)$.

§ 10.

In linea recta A O, tanquam axe communi, dispositæ intelligantur lentes quotcunque P, P^I, P^{II}, P^{III}, &c. quarum foci conjugati sint respective, lentis
V o l. L I. 6 G primæ

primæ A et $\overset{I}{A}$; secundæ $\overset{I}{A}$ et $\overset{II}{A}$; tertiæ $\overset{II}{A}$ et $\overset{III}{A}$;
quartæ $\overset{III}{A}$ et $\overset{IV}{A}$, &c.



Radius luminis, veniens à puncto A, refringatur in omnibus his lentibus in punctis $\overset{I}{L}$, $\overset{II}{L}$, $\overset{III}{L}$, $\overset{IV}{L}$, &c. pergens secundum cursum $A \overset{I}{L} \overset{II}{L} \overset{III}{L} \overset{IV}{L}$, et post singulas refractiones occurrat axi A O in punctis $\overset{I}{B}$, $\overset{II}{B}$, $\overset{III}{B}$, $\overset{IV}{B}$, &c. aberrans à focis $\overset{I}{A}$, $\overset{II}{A}$, $\overset{III}{A}$, $\overset{IV}{A}$, &c. longitudinibus $\overset{I}{A} \overset{I}{B}$, $\overset{II}{A} \overset{II}{B}$, $\overset{III}{A} \overset{III}{B}$, $\overset{IV}{A} \overset{IV}{B}$, &c. Quærentur jam hæ aberrationes.

Lentium $\overset{I}{P}$, $\overset{II}{P}$, $\overset{III}{P}$, $\overset{IV}{P}$, &c. distantie focorum principalium dicantur respective $\overset{I}{P}$, $\overset{II}{P}$, $\overset{III}{P}$, $\overset{IV}{P}$, &c. et indices $\overset{I}{x}$, $\overset{II}{x}$, $\overset{III}{x}$, $\overset{IV}{x}$, &c. Distantie focorum conjugatorum à suis respective lentibus sint: pro lente prima $\overset{I}{P}$, $\overset{I}{P} \overset{I}{A} = \overset{I}{A}$, $\overset{I}{P} \overset{I}{A} = \overset{I}{B}$; pro lente secunda $\overset{II}{P}$, $\overset{II}{P} \overset{II}{A} = \overset{II}{A}$, $\overset{II}{P} \overset{II}{A} = \overset{II}{B}$; pro lente tertia $\overset{III}{P}$, $\overset{III}{P} \overset{III}{A} = \overset{III}{A}$, $\overset{III}{P} \overset{III}{A} = \overset{III}{B}$; pro lente quarta $\overset{IV}{P}$, $\overset{IV}{P} \overset{IV}{A} = \overset{IV}{A}$, et $\overset{IV}{P} \overset{IV}{A} = \overset{IV}{B}$, et ita porro: ponatur præterea $\overset{I}{P} \overset{I}{L}$, five distantia puncti incidentiæ in primam lentem ab axe = $\overset{I}{L}$, et erit $\overset{I}{P} \overset{I}{L} = \frac{\overset{I}{A}}{\overset{I}{B}} \times \overset{I}{L}$, $\overset{II}{P} \overset{II}{L} = \frac{\overset{II}{A} \cdot \overset{II}{A}}{\overset{II}{B} \cdot \overset{II}{B}} \times \overset{II}{L}$, $\overset{III}{P} \overset{III}{L} = \frac{\overset{III}{A} \cdot \overset{III}{A} \cdot \overset{III}{A}}{\overset{III}{B} \cdot \overset{III}{B} \cdot \overset{III}{B}} \times \overset{III}{L}$, et ita porro.

I. Vidimus in § 9. aberrationem $\overset{I}{A}B$ lentis primæ P effe $\frac{B^2 L^2}{2g \cdot P^3} \times \left(\frac{P}{\overset{I}{A} + \overset{I}{B}} + f + g^2 x^2 \right)$, ubi $f = \frac{4i - r \cdot r}{4 \cdot i - r^2}$, et $g = \frac{i + 2r}{i}$.

II. Concipiatur à foco secundæ lentis $\overset{II}{A}$ ipsi $\overset{I}{A}$ conjugato in illam incidere radius $\overset{II}{A}L$, qui refractus secundum $\overset{I}{L}C$, conveniat cum axe in C, aberrans à foco $\overset{I}{A}$ longitudine $\overset{I}{A}C$: et erit per § 9. aberratio $\overset{I}{A}C = \frac{\overset{I}{A}^2 L^2}{2g \cdot B^2 \cdot P^3} \times \left(\frac{\overset{I}{P}}{\overset{I}{A} + \overset{I}{B}} + f + g^2 x^2 \right)$, scribendo

scilicet $\frac{\overset{I}{A}}{B} \times L$ pro L, $\overset{I}{B}$ pro A, $\overset{I}{A}$ pro B, et P pro

P. Hæc aberratio $\overset{I}{A}C$, addita ad aberrationem primæ lentis modo inventam, $\overset{I}{A}B$, dat summam BC. Est

vero BC ad $\overset{II}{B}\overset{II}{A}$ ut $\overset{I}{P}\overset{I}{A}q$ ad $\overset{II}{P}\overset{II}{A}q$, five $\overset{I}{A}^2$ ad $\overset{II}{B}^2$, id est quantitates aberrationum à focis conjugatis sunt ut quadrata distantiarum focorum à lente, quod facile probari potest. Ergo aberratio $\overset{II}{A}\overset{I}{B}$, à refractione

per duas lentes P et $\overset{I}{P}$ producta, habetur $= \frac{\overset{II}{B}^2}{\overset{I}{A}^2} \times BC$

$$= \frac{\overset{II}{B}^2 L^2}{2g} \times \left[\frac{B^2}{\overset{I}{A}^2 \cdot P^3} \left(\frac{P}{\overset{I}{A} + \overset{I}{B}} + f + g^2 x^2 \right) + \frac{\overset{I}{A}^2}{B^2 \cdot P^3} \left(\frac{\overset{I}{P}}{\overset{I}{A} + \overset{I}{B}} + f + g^2 x^2 \right) \right].$$

III. Concipiatur fimiliter à foco interiori $\overset{\text{III}}{\text{A}}$ lentis tertiae $\overset{\text{II}}{\text{P}}$ incidere in illam radium $\overset{\text{III}}{\text{A}} \overset{\text{II}}{\text{L}}$, cumque refractum convenire cum axe in puncto $\overset{\text{I}}{\text{C}}$, et aberrare à foco exteriori $\overset{\text{II}}{\text{A}}$ longitudine $\overset{\text{II}}{\text{A}} \overset{\text{I}}{\text{C}}$, quæ per § 9. est

$$\frac{\overset{\text{I}}{\text{A}}^2 \cdot \overset{\text{II}}{\text{A}}^2 \cdot \overset{\text{I}}{\text{L}}^2}{2g \cdot \overset{\text{I}}{\text{B}}^2 \cdot \overset{\text{I}}{\text{B}}^2 \cdot \overset{\text{II}}{\text{P}}^3} \times \left(\frac{\overset{\text{II}}{\text{P}}}{\overset{\text{II}}{\text{A}} + \overset{\text{II}}{\text{B}}} + f + g^2 \overset{\text{II}}{\text{x}}^2 \right). \text{ Hæc addita ad aberrationem duarum priorum lentium modo inventam, } \overset{\text{II}}{\text{A}} \overset{\text{I}}{\text{B}}, \text{ dat summam } \overset{\text{II}}{\text{A}} \overset{\text{I}}{\text{B}} + \overset{\text{II}}{\text{A}} \overset{\text{I}}{\text{C}} = \overset{\text{I}}{\text{B}} \overset{\text{I}}{\text{C}}, \text{ deinde fiat ut } \overset{\text{II}}{\text{A}}^2 \text{ ad } \overset{\text{II}}{\text{B}}^2; \text{ ita hæc summa } \overset{\text{I}}{\text{B}} \overset{\text{I}}{\text{C}} \text{ ad aberrationem trium lentium } \overset{\text{III}}{\text{A}} \overset{\text{II}}{\text{B}} = \frac{\overset{\text{II}}{\text{B}}^2 \cdot \overset{\text{I}}{\text{L}}^2}{2g} \times \left[\frac{\overset{\text{I}}{\text{B}}^2 \cdot \overset{\text{I}}{\text{B}}^2}{\overset{\text{II}}{\text{A}}^2 \cdot \overset{\text{I}}{\text{A}}^2 \cdot \overset{\text{II}}{\text{P}}^3} \left(\frac{\overset{\text{I}}{\text{P}}}{\overset{\text{I}}{\text{A}} + \overset{\text{I}}{\text{B}}} + f + g^2 \overset{\text{I}}{\text{x}}^2 \right) + \frac{\overset{\text{I}}{\text{B}}^2 \cdot \overset{\text{II}}{\text{A}}^2}{\overset{\text{I}}{\text{B}}^2 \cdot \overset{\text{II}}{\text{A}}^2 \cdot \overset{\text{II}}{\text{P}}^3} \left(\frac{\overset{\text{II}}{\text{P}}}{\overset{\text{II}}{\text{A}} + \overset{\text{II}}{\text{B}}} + f + g^2 \overset{\text{II}}{\text{x}}^2 \right) + \frac{\overset{\text{I}}{\text{A}}^2 \cdot \overset{\text{II}}{\text{A}}^2}{\overset{\text{I}}{\text{B}}^2 \cdot \overset{\text{I}}{\text{B}}^2 \cdot \overset{\text{II}}{\text{P}}^3} \left(\frac{\overset{\text{II}}{\text{P}}}{\overset{\text{II}}{\text{A}} + \overset{\text{II}}{\text{B}}} + f + g^2 \overset{\text{II}}{\text{x}}^2 \right) \right].$$

IV. Eadem ratione invenitur aberratio $\overset{\text{IV}}{\text{A}} \overset{\text{III}}{\text{B}}$ post refractionem per quatuor lentes $\overset{\text{III}}{\text{P}}, \overset{\text{II}}{\text{P}}, \overset{\text{I}}{\text{P}}, \overset{\text{III}}{\text{P}}, = \frac{\overset{\text{III}}{\text{B}}^2 \cdot \overset{\text{I}}{\text{L}}^2}{2g} \times \left[\frac{\overset{\text{I}}{\text{B}}^2 \cdot \overset{\text{I}}{\text{B}}^2 \cdot \overset{\text{II}}{\text{A}}^2}{\overset{\text{III}}{\text{A}}^2 \cdot \overset{\text{II}}{\text{A}}^2 \cdot \overset{\text{I}}{\text{A}}^2 \cdot \overset{\text{III}}{\text{P}}^3} \times \left(\frac{\overset{\text{I}}{\text{P}}}{\overset{\text{I}}{\text{A}} + \overset{\text{I}}{\text{B}}} + f + g^2 \overset{\text{I}}{\text{x}}^2 \right) + \frac{\overset{\text{I}}{\text{B}}^2 \cdot \overset{\text{II}}{\text{A}}^2 \cdot \overset{\text{I}}{\text{L}}^2}{\overset{\text{I}}{\text{B}}^2 \cdot \overset{\text{II}}{\text{A}}^2 \cdot \overset{\text{III}}{\text{P}}^3} \times \left(\frac{\overset{\text{II}}{\text{P}}}{\overset{\text{II}}{\text{A}} + \overset{\text{II}}{\text{B}}} + f + g^2 \overset{\text{II}}{\text{x}}^2 \right) + \frac{\overset{\text{I}}{\text{A}}^2 \cdot \overset{\text{II}}{\text{A}}^2 \cdot \overset{\text{III}}{\text{A}}^2}{\overset{\text{I}}{\text{B}}^2 \cdot \overset{\text{I}}{\text{B}}^2 \cdot \overset{\text{II}}{\text{P}}^3} \left(\frac{\overset{\text{III}}{\text{P}}}{\overset{\text{III}}{\text{A}} + \overset{\text{III}}{\text{B}}} + f + g^2 \overset{\text{III}}{\text{x}}^2 \right) \right]. \text{ Et eadem ratione progredi licet ad definiendam}$

definiendam aberrationem radiorum post transitum per plures lentes.

In his computationibus omnes lentes concipiuntur convexæ, et foci radiorum incidentium ponuntur jacere ante lentes suas, refractorum vero post lentes respectu cursus radiorum, quemadmodum figura exhibet. Formulæ autem allatæ ad quosvis alios casus facile accommodantur, mutando signa symbolorum, prout requirit mutatio hypotheseos, quod monuisse sufficiat.

§ 11.

Datis positionibus et focorum principalium distantis lentium quotcunque, oporteat invenire formas earum, quæ efficiant, ut radii luminis, à dato in communi earum axe puncto advenientes, post refractionem colligantur in ultimo foco sine aberratione, si fieri potest.

Solutio problematis generaliter absolvitur, ponendo expressionem aberrationis, datæ multitudini lentium convenientem, (§ 10.) nihilo æqualem, quo ipso habetur æquatio definiens rationem requisitam indicum

singularum lentium x , x^I , x^{II} , x^{III} , &c. Et quoniam relatio omnium indicum unica hac æquatione continetur, patet multum hic locum esse arbitrariæ nonnullorum assumptioni, prout requirunt vel suadent aperturæ lentium aliæque circumstantiæ ex scopo instrumenti dijudicandæ, de quibus hic non est sermo. In genere tantum cavendum est, ne assumptio ita fiat, ut aliquis indicum prodeat imaginarius.

Determinatis sic indicibus x , x^I , x^{II} , x^{III} , &c. inveniuntur radii facierum per § 8. Dicatur scilicet radius faciei

faciei anterioris singularum lentium a , posterioris b ; distantia foci principalis à lente, P ; distantia foci radiorum incidentium ante lentem, A ; distantia foci radiorum refractorum post lentem, B ; et index ad lentem pertinens, x ; et erit $\frac{1}{a} = \frac{b}{B} + \frac{K}{A} + \frac{x}{P}$, et $\frac{1}{b} = \frac{b}{A} + \frac{K}{B} - \frac{x}{P}$, ubi $b = \frac{i \cdot 2i + r}{2 \cdot i - r \cdot i + 2r}$, et $K = \frac{4r^2 + ri - 2i^2}{2 \cdot i - r \cdot i + 2r}$, unde dabuntur singularum lentium radii a et b .

Si nullus detur valor realis indicum x , x^I , x^{II} , x^{III} , &c. id quod accidit, si omnes æquationis termini ad unam partem rejecti fuerint ejusdem signi, casus propositus est impossibilis.

§ 12.

EXEMPLUM I. Ex duabus lentibus telescopium componere, quod sit liberum ab aberratione, et objecta visa amplifcet in data ratione n ad 1. Abstrahitur autem hic a reliquis boni telescopii requisitis.

Lentes duæ primæ P et $\overset{I}{P}$ repræsentent binas lentes telescopii quæfiti, P quidem objectivam et $\overset{I}{P}$ ocularem (vid. fig. § 10.). Aberratio harum duarum lentium (§ 10. cas. 11.) nihilo æqualis posita dat æquationem generalem: $g^2 \times \left(\frac{B^2 \cdot x^2}{\overset{I}{A}^2 \cdot P^3} + \frac{\overset{I}{A}^2 \cdot x^2}{B^2 \cdot \overset{I}{P}^3} \right) + f \times \left(\frac{B^2}{\overset{I}{A}^2 \cdot P^3} + \frac{\overset{I}{A}^2}{B^2 \cdot \overset{I}{P}^3} \right) + \frac{B^2}{\overset{I}{A}^2 \cdot P^2 \cdot A + B} + \frac{\overset{I}{A}^2}{B^2 \cdot \overset{I}{P}^2 \cdot \overset{I}{A} + \overset{I}{B}} = 0$. Quoniam per constitutionem telescopii, radii lu-

minis

minis in lentem objectivam incidentes, et è lente oculari emergentes, sunt axi telescopii paralleli, erit A infinita, adeoque $B = P$; item $\overset{I}{B}$ infinita, adeoque $\overset{I}{A} = \overset{I}{P}$. Quare deletis duobus ultimis æquationis terminis, utpote evanescentibus, et in reliquis in scribendo P et $\overset{I}{P}$ pro B et $\overset{I}{A}$, habetur pro casu hujus exempli æquationis $g^2 \times (P x^2 + \overset{I}{P} x^2) + f \times (P + \overset{I}{P}) = 0$, relationem indicum x et $\overset{I}{x}$ definiens, requisitam ad id, ut aberratio radiorum è lente oculari emergentium nulla sit. Jam si in hac æquatione P et $\overset{I}{P}$ ponantur ejusdem signi, sive utraque lens convexa, vel utraque concava, nullus valor realis haberi potest pro x et $\overset{I}{x}$, adeoque problema in hoc casu erit impossibile. Statuatur itaque $\overset{I}{P}$ negativa, sive lens ocularis concava, sumendo pro exponente amplificationis objecti telescopio visi, n , numerum quemvis positivum, et faciendo $\overset{I}{P} = -\frac{1}{n}P$, ut patet ex opticis. Scripto hoc valore pro $\overset{I}{P}$ in æquatione indicum, evadit illa $g^2 \times (x^2 - nx^2) = f \times n - 1$. Horum itaque indicum utrumlibet pro lubitu assumere licet, modo ne $\overset{I}{x}$ fiat minor, quam $\frac{\sqrt{n-1} \cdot f}{g}$, quod redderet indicem x imaginarium. Altero indicum sic assumpto, alter determinatur per allatam æquationem $g^2 \times (x^2 - nx^2) = f \cdot \overline{n-1}$. Quo facto radii facierum utriusque lentis prodeunt per § 11. ut sequitur:

$$\begin{aligned} \text{Radius faciei anterioris lentis objectivæ} &= \frac{P}{b+x}, \\ \text{posterioris} &= \frac{P}{K-x}, \end{aligned}$$

$$\begin{aligned} \text{Radius faciei anterioris lentis ocularis} &= \frac{p}{K+x}, \\ \text{posterioris} &= \frac{\overset{I}{P}}{b-x}, \text{ ubi } \overset{I}{P} \text{ sumenda est negativa, sive} \\ \overset{I}{P} &= -\frac{1}{n}P, \text{ ut dictum est.} \end{aligned}$$

Pro quovis itaque pari valorum correspondentium indicum x et $\overset{I}{x}$, quatuor prodeunt diversæ problematis solutiones, ob signa duplicia ipsarum x et $\overset{I}{x}$, unde duæ prodeunt formæ utriusque lentis, quarum quælibet combinari potest cum binis formis alterius lentis.

Si detur vel assumatur forma alterutrius lentis, forma alterius per supradicta commode determinatur. Sit v. gr. lens objectiva plano-convexa, habens faciem convexam antrorsum versam, planam retrorsum.

Radius faciei ejus posterioris $\frac{P}{K-x}$ statuatur infinitus, et erit $x = K$, adeoque radius faciei, anterioris $\frac{P}{b+x} = \frac{P}{b+K} = \frac{i-r}{r} \times P$. Pro x scribatur K in æquatione indicum $\overset{I}{x}^2 - nx^2 = \frac{n-1 \cdot f}{g^2}$, et erit $\overset{I}{x} = \sqrt{\frac{n-1 \cdot f}{g^2} + nK^2}$, unde dantur radii facierum lentis ocularis, idque dupliciter, ob signum duplex valoris ipsius $\overset{I}{x}$.

Vidimus

Vidimus in solutione hujus problematis, aberrationem à duabus lentibus productam non posse evanescere, si utraque fuerit convexa, vel utraque concava, cujuscumque sint formæ et quomodocunque componentur. Idem quoque obtinet in quacunque lentium multitudine, quæ omnes sint convexæ, vel omnes concavæ. Omnes enim lentes, quarum facies superficierum sphaëricarum sunt segmenta, aberrationes a focus suis eo producant, quod radios luminis refractione nimium inflectunt; unde facile perspicitur, lentem convexam, convexis additam, vel concavam concavis, errores a prioribus productos augere. Quare ut aberratio a foco ultimo evanescat, debent lentium aliæ esse convexæ, aliæ concavæ, quoniam radiorum incurvationes in unam partem corrigantur per nimias incurvationes factas in partem contrariam.

§ 13.

EXEMPLUM II. Propositum sit investigare formas binarum lentium, quæ juxta se positæ radios incidentes axi parallelos in data à lentibus distantia R colligant, citra aberrationem ex figura facierum sphaërica oriundam.

Quoniam in casu hujus exempli radii incidentes ponuntur paralleli erit in æquatione (§ 10. cas. 2.) pro relatione binorum indicum x et x' , A infinita, adeoque $B = P$. Et quoniam lentes ponuntur juxta se positæ, erit earum distantia $B + A = 0$, sive $A = -B = -P$. Præterea est etiam $\frac{1}{B} = \frac{1}{P} + \frac{1}{P}$, unde $\frac{1}{B} - P = -\frac{P^2}{P + P}$. Substitutis itaque his

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valoribus,

valoribus, erit æquatio pro relatione indicum in casu præfenti: $g^2 \cdot (\dot{P}^3 \cdot x^2 + P^3 \cdot \dot{x}^2) + f \cdot \overline{P^3 + \dot{P}^3} = P \cdot \dot{P} \cdot \overline{P + \dot{P}}$, five $P^3 x^2 + \dot{P}^3 \dot{x}^2 = \frac{P + \dot{P}}{g^2} \times (\overline{1 + f} \cdot P \dot{P} - f \cdot P^2 + \dot{P}^2)$, quæ assumpto numero quovis n , et statuendo $\dot{P} = nP$, evadit $x^2 + n^3 \dot{x}^2 = \frac{n + 1}{g^2} \times (n - \overline{n^2 - n + 1} \cdot f)$. Ne autem casus propositus fiat impossibilis, ob utramque lentem convexam, vel utramque concavam, per n semper intelligi debet numerus negativus. Assumatur jam pro Cubitu alteruter indicum x et $\dot{x}$, et dabitur alter per æquationem allatam.

Determinato vero utroque indice, radii facierum erunt ut sequitur:

Radius faciei anterioris lentis primæ $= \frac{P}{b + x}$,
posterioris $= \frac{P}{K - x}$;

Radius faciei anterioris lentis secundæ $= \frac{\dot{P}}{n + 1 \cdot b - nK + x}$, posterioris $= \frac{\dot{P}}{n + 1 \cdot K - nb - x}$,

ubi est $nP = \dot{P}$, posito n numero negativo.

Distantia foci principalis systematis binarum lentium P et $\dot{P}$ dicatur R , et quoniam est $\frac{1}{R} = \frac{1}{P} + \frac{1}{\dot{P}}$, erit $P = \frac{n + 1}{n} R$, et $\dot{P} = \overline{n + 1} \cdot R$, quare in expressionibus radiorum scribendo hos valores pro P et $\dot{P}$, habebuntur:

$$\text{Radius faciei anterioris lentis primæ} = \frac{\frac{n+1}{n} R}{b+x},$$

$$\text{posterioris} = \frac{\frac{n+1}{n} R}{K-x};$$

$$\text{Radius faciei anterioris lentis secundæ} = \frac{\frac{n+1}{n+1} \cdot R}{n+1 \cdot b - nK + x}, \text{ posterioris} = \frac{\frac{n+1}{n+1} \cdot R}{n+1 \cdot K - nb - x}.$$

Et hæ lentes juxta se positæ radios incidentes axi parallelas in distantia à lente, R , colligent sine aberratione, si distantia foci lentis concavæ major fuerit distantia foci lentis convexæ; si autem contra dispergentur radii sine aberratione à puncto ante lentem, cujus distantia positive sumta est R .

Pro casu singulari hujus exempli, ponamus requiri formas binarum lentium ita comparatarum, ut distantia foci principalis lentis concavæ sit ad distantiam foci principalis lentis convexæ, ut 3 ad 2, et ut radii colligantur in distantia R à lentibus. Si lens anterior debet esse convexa, erit $n = -\frac{3}{2}$, et æquatio definiens relationem indicum lentis convexæ x et lentis concavæ x' , $8x'^2 - 27x^2 = \frac{19f+6}{g^2}$. De-

terminatis vero decenter indicibus x et x' , radii facierum erunt, ut sequitur:

$$\text{Radius faciei anterioris lentis primæ} = \frac{R}{3b+3x},$$

$$\text{posterioris} = \frac{R}{3K-3x};$$

$$\text{Radius faciei anterioris lentis secundæ} = \frac{R}{3K-b+2x}, \text{ posterioris} = \frac{R}{3b-K-2x'}.$$

Si vero lens anterior debet esse concava, erit $n = -\frac{2}{3}$, et ratio indicum $8x^2 - 27x^2 = \frac{19f+6}{g^2}$, radii vero facierum ut sequitur:

$$\text{Radius faciei anterioris lentis primæ} = -\frac{R}{2b+2x},$$

$$\text{posterioris} = -\frac{R}{2K-2x};$$

$$\text{Radius faciei anterioris lentis secundæ} = \frac{R}{b+2K+3x},$$

$$\text{posterioris} = \frac{R}{K+2b-3x}.$$

§ 14.

EXEMPLUM III. Propositum sit, telescopium ex tribus lentibus componere, quarum prima P et secunda P^I juxta se positæ vitrum objectivum duplex constituent, cujus distantia foci principalis sit R ; tertia vero five ocularis P^{II} , habeat distantiam foci $P^{II} = \frac{R}{m}$, ut potentia amplificandi designetur per numerum m . Ponatur præterea $P^I = nP$, intelligendo per n numerum datum negativum, eumque unitate majorem, si lens convexa anterior est collocanda, minorem si posterior, ut radii post refractionem in vitro objectivo convergant, quemadmodum expositum est in exemplo præcedente. Quærentur autem formæ harum trium lentium, ut telescopium liberum sit ab omni erratione ex figura illarum sphærica oriunda.

Aberratio à tribus lentibus P, P^I, P^{II} , oriunda (§ 10. cas. III.) nihilo æqualis posita, dat $\frac{B^2 \cdot B^2}{A^2 \cdot A^2 \cdot P^3} \left(\frac{P}{A+B} \right)$

+

$$+ f + g^2 x^2) + \frac{\overset{I}{B^2} \cdot \overset{I}{A^2}}{\overset{I}{B^2} \cdot \overset{II}{A^2} \cdot \overset{I}{P^3}} \left(\frac{\overset{I}{P}}{\overset{I}{A} + \overset{I}{B}} + f + g^2 x^2 \right) \\ + \frac{\overset{I}{A^2} \cdot \overset{II}{A^2}}{\overset{I}{B^2} \cdot \overset{I}{B^2} \cdot \overset{II}{P}} \left(\frac{\overset{II}{P}}{\overset{II}{A} + \overset{II}{B}} + f + g^2 x^2 \right) = 0.$$

Symbola hanc æquationem ingredientia per hypothefin hujus exempli ita determinantur: ob A infinitam est $\frac{\overset{I}{P}}{\overset{I}{A} + \overset{I}{B}} = 0$, et $B = P$. Ob $\overset{II}{B}$ infinitam est $\frac{\overset{II}{P}}{\overset{II}{A} + \overset{II}{B}} = 0$, et $\overset{II}{A} = \overset{II}{P}$. Ob lentes P et $\overset{I}{P}$ juxta se positas; est earum distantia $\overset{I}{A} + B = 0$, five $\overset{I}{A} = -B = -P$. Ob $\frac{\overset{I}{R}}{\overset{I}{R}} = \frac{\overset{I}{P}}{\overset{I}{P}} + \frac{\overset{I}{P}}{\overset{I}{P}}$, et $\overset{I}{P} = nP$, est $P = \frac{n + \overset{I}{I}}{n} R$, et $\overset{I}{P} = \frac{n + \overset{I}{I}}{n + \overset{I}{I}} \cdot R$. Præterea est $\overset{I}{B} = R$, et $\overset{II}{P} = \frac{\overset{II}{R}}{m}$. His itaque valoribus substitutis, et æquatione ordinata, habetur relatio indicum $x, \overset{I}{x}, \overset{II}{x}$, expressa ut sequitur: $g^2 \times (n^3 x^2 + \overset{I}{x}^2 + \frac{n + \overset{I}{I}^3}{m} \overset{II}{x}^2) + f \times (n^3 + 1 + \frac{n + \overset{I}{I}^3}{m}) = n \cdot \frac{n + \overset{I}{I}}{n + \overset{I}{I}}$. Determinatis indicibus $x, \overset{I}{x}, \overset{II}{x}$, habentur per § II. radii facierum: videlicet,

$$\text{Radius faciei anterioris lentis primæ} = \frac{\frac{n + \overset{I}{I}}{n + \overset{I}{I}} \cdot R}{n \cdot \overset{I}{b} + x}, \\ \text{posterioris} = \frac{\frac{n + \overset{I}{I}}{n + \overset{I}{I}} \cdot R}{n \cdot \overset{II}{K} - x};$$

$$\text{Radius faciei anterioris lentis secundæ} = \frac{\frac{n + \overset{I}{I}}{n + \overset{I}{I}} \cdot R}{\frac{n + \overset{I}{I}}{n + \overset{I}{I}} \cdot \overset{I}{b} - nK + x}, \text{ posterioris} = \frac{\frac{n + \overset{I}{I}}{n + \overset{I}{I}} \cdot R}{\frac{n + \overset{I}{I}}{n + \overset{I}{I}} \cdot \overset{II}{K} - nb - x};$$

Radius

$$\begin{aligned} \text{Radius faciei anterioris lentis tertiæ} &= \frac{R}{m \cdot K + x^{\frac{11}{2}}}, \\ \text{posterioris} &= \frac{R}{m \cdot b - x^{\frac{11}{2}}}. \end{aligned}$$

Ita si vitrum objectivum hujus telescopii componendum fit ex duabus lentibus, convexa et concava, quarum distantiae focales sint ut 3 et 2, debeatque primo lens convexa anterior collocari, oportebit sumere $n = -\frac{3}{2}$, et erit æquatio pro indicibus:

$$g^2 \times (27x^2 - 8x^{\frac{1}{2}} + \frac{x^2}{m}) + f \times (19 + \frac{1}{m}) + 6 = 0, \text{ et radii facierum:}$$

$$\begin{aligned} \text{Radius faciei anterioris lentis primæ} &= \frac{R}{3b + 3x}, \\ \text{posterioris} &= \frac{R}{3K + 3x}; \end{aligned}$$

$$\begin{aligned} \text{Radius faciei anterioris lentis secundæ} &= \\ -\frac{R}{3K - b + 2x}, \text{ posterioris} &= -\frac{R}{3b - K - 2x}; \end{aligned}$$

$$\begin{aligned} \text{Radius faciei anterioris lentis tertiæ} &= \frac{R}{mK + x^{\frac{11}{2}}}, \\ \text{posterioris} &= \frac{R}{mb - mx^{\frac{11}{2}}}. \end{aligned}$$

Si vero lens concava anterior est collocanda, debet sumi $n = -\frac{2}{3}$, unde æquatio pro indicibus prodit

$$g^2 \times (-8x^2 + 27x^{\frac{1}{2}} + \frac{x^2}{m}) + f \times (19 + \frac{1}{m}) + 6 = 0, \text{ et consequenter:}$$

$$\begin{aligned} \text{Radius faciei anterioris lentis primæ} &= -\frac{R}{2b + 2x}, \\ \text{posterioris} &= -\frac{R}{2K - 2x}; \end{aligned}$$

Radius

$$\text{Radius faciei anterioris lentis secundæ} = \frac{R}{b + 2K + 3x}, \text{ posterioris} = \frac{R}{K + 2b - 3x};$$

$$\text{Radius faciei anterioris lentis tertiæ} = \frac{R}{mK + m'x}$$

$$\text{posterioris} = \frac{R}{mb - m'x}.$$

§ 15.

Vidimus in solutione generati novissimi problematis (§ ii.) et subjunctis exemplis (§ 12, 13, 14.) relationem indicum x , x^I , x^{II} , &c. unica tantum æquatione definiri, adeoque judicio artificis plurimum esse relictum in commoda eorum assumptione facienda. Duæ autem sunt considerationes, quibus hæc assumptio utcunque dirigi possit. Prima est, ut nullus indicum fiat nimis magnus, five, quod eodem recidit, ut formæ lentium quamproxime accedant ad eas, quæ in datis circumstantiis aberrationem dant omnium minimam. Hac enim ratione errores in forma lentium forte commissi minus nocebunt, quemadmodum ex natura minimi notum est. Altera est, ut singulæ lentes, quam fieri potest, proxime evadant æqualiter utrinque convexæ, vel æqualiter concavæ, quo fiet, ut majores aperturæ sint patientes. Quantitates enim aberrationum supra definitæ non sunt nisi quamproxime veræ, et tanto magis fallunt, quanto facies lentium fuerint majora suarum sphærarum segmenta. Binæ hæ regulæ, etiamsi sibi invicem sæpius adversantur, nec facile definiri queat, quantum in quovis dato casu uni vel alteri ipsarum sit tribuendum; non dubito tamen, quin judicioso artifice sint profuturæ

profuturæ ad mediam viam tenendam inter utramque.

Eædem quoque considerationes impediunt, quo minus certi aliquid definire auserim, de maximo possibili effectu instrumentorum opticorum secundum hæc principia compositorum, vel de modo perfectissima componendi. Regulæ huc pertinentes tutissime expectantur ab experientia, modo dirigatur à theoria.

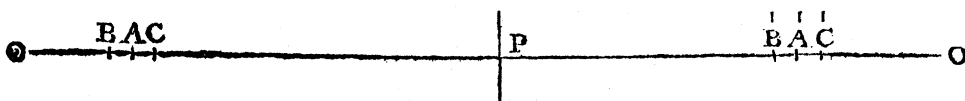
§ 16.

Expositis jam breviter quæ dicenda habuimus de aberrationibus radiorum homogeneorum refractorum, ortis a lentium figura sphærica, ut et de modo illas corrigendi in instrumentis opticis: liceat etiam pauca addere de altera illa aberrationis specie, quæ a diversa radiorum refrangibilitate oritur, quæ objecta trans lentes visâ coloribus inficit, et maximam confusionis partem producit. Creditum hucusque communiter fuit hoc vitium nulla arte emendabile, donec ingeniosissimus artifex Londinensis, Dollandus, experimentis institutis feliciter deprehenderet, varia dari vitrorum genera, quæ, licet fere æqualiter refringant certum quoddam radiorum genus, plurimum tamen discrepent viribus refringendi reliqua radiorum genera, adeoque et radios heterogeneous a se invicem separandi: quo eximio invento et theoriâ et praxim optices insigniter auxit. Discimus enim hinc, rationes refractionem radiorum diversi generis nullo modo a se invicem dependere, ut hucusque fuit creditum; adeoque frustra quæri regulam, qua ex datis refractionibus radiorum diversi generis in uno aliquo medio inveniantur refractiones eorundem in alio;
vel

vel qua ex data refractione unius alicujus radii in quovis medio, inveniuntur refractiones reliquorum in eodem medio. Praxis quoque telescopiorum dioptricorum insigne incrementum hujus inventi beneficio nacta est. Scilicet sagacissimus inventor modum inde derivavit corrigendi aberrationes ex separatione radiorum heterogeneorum oriundas, componendo vitrum objectivum ex duabus lentibus, una convexa, altera concava, quæ ita sint comparatæ, ut una alterius effectum in radiis separandis destruat, quemadmodum in Vol. L. Part. II. p. 733. et seq. Transactionum Philosophicarum exposuit. Quod artificium ita breviter explicabimus.

§ 17.

Sit P lens quæcunque, habens radios facierum suarum a et b ; distantiam foci principalis mediocriter refrangibilem P ; et $a \propto$ in rectam OO .



Ratio refractionis radiorum mediocriter refrangibilem ex aëre in lentem sit N ad 1 ; radiorum maxime refrangibilem $N + n$ ad 1 , et radiorum minime refrangibilem $N - n$ ad 1 , existente n numero valde parvo respectu ipsius N . In axe lentis OO sint puncta A, B, C foci radiorum in lentem P incidentium, A mediocriter refrangibilem, B maxime et C minime refrangibilem, quorum focorum distantiae AB, AC sint valde parvæ, et ponantur puncta

$\overset{1}{A}, \overset{1}{B}, \overset{1}{C}$, foci conjugati ipsorum A, B, C respective,

pro sua quique radiorum specie, ita ut sint $BC, \overset{1}{B}\overset{1}{C}$,

diffipationes radiorum heterogeneorum circa focos conjugatos A et $\overset{I}{A}$. Quærat^r earum relatio.

Quoniam B et $\overset{I}{B}$ sunt foci conjugati radiorum maxime refrangibilium, quorum ratio refractionis est $N + n$ ad 1, erit $\frac{I}{PB} + \frac{I}{P\overset{I}{B}} = \overline{N + n - 1} \cdot \overline{\frac{I}{a} + \frac{I}{b}}$. Similiter quoniam puncta C et $\overset{I}{C}$ sunt foci conjugati radiorum minime refrangibilium, quorum ratio refractionis est $N - n$ ad 1, erit $\frac{I}{PC} + \frac{I}{P\overset{I}{C}} = \overline{N - n + 1} \cdot \overline{\frac{I}{a} + \frac{I}{b}}$. Subtrahatur hæc posterior æquatio à priore, et habebitur $\frac{I}{PB} - \frac{I}{PC} + \frac{I}{P\overset{I}{B}} - \frac{I}{P\overset{I}{C}} = 2n \cdot \overline{\frac{I}{a} + \frac{I}{b}}$, five conjungendo binos terminos: $\frac{PC - PB}{PB \times PC} + \frac{P\overset{I}{C} - P\overset{I}{B}}{P\overset{I}{B} \times P\overset{I}{C}} = 2n \cdot \overline{\frac{I}{a} + \frac{I}{b}}$, quæ pro $PB \times PC$ et $P\overset{I}{B} \times P\overset{I}{C}$, scribendo PAq et $P\overset{I}{A}q$, et pro $\frac{I}{a} + \frac{I}{b}$ scribendo $\frac{I}{N - 1 \cdot P}$, evadit $\frac{BC}{PAq} = \frac{2n}{N - 1 \cdot P} + \frac{BC}{P\overset{I}{A}q}$, exhibens relationem inter diffipationes conjugatas BC et $\overset{I}{B}\overset{I}{C}$.

His præmissis, intelligantur lentes quocunque P, $\overset{I}{P}$, $\overset{II}{P}$, $\overset{III}{P}$, &c. quarum distantiae focorum principalium respectu radiorum mediocriter refrangibilium sint P, $\overset{I}{P}$, $\overset{II}{P}$, $\overset{III}{P}$, &c. respective, ordine dispositæ in axe earum

item per P, N et n eorum valores singulis lentis proprii, habebuntur tot æquationes, quot sunt lentes:

$$\text{videlicet; } \overset{I}{B}\overset{I}{C} = B^2 \times \left(\frac{2n}{\overset{I}{N} - \overset{I}{I} \cdot \overset{I}{P}} + \frac{BC}{A^2} \right); \overset{II}{B}\overset{II}{C} = \overset{I}{B}^2 \times \left(\frac{2n}{\overset{II}{N} - \overset{II}{I} \cdot \overset{II}{P}} + \frac{\overset{I}{B}\overset{I}{C}}{A^2} \right); \overset{III}{B}\overset{III}{C} = \overset{II}{B}^2 \times \left(\frac{2n}{\overset{III}{N} - \overset{III}{I} \cdot \overset{III}{P}} + \frac{\overset{II}{B}\overset{II}{C}}{A^2} \right); \overset{IV}{B}\overset{IV}{C} = \overset{III}{B}^2 \times \left(\frac{2n}{\overset{IV}{N} - \overset{IV}{I} \cdot \overset{IV}{P}} + \frac{\overset{III}{B}\overset{III}{C}}{A^2} \right); \text{ et ita}$$

porro, si plures fuerint lentes. Ponantur puncta B, A, C, coincidere, five ex primo foco A prodire radium compositum, ut dissipatio prima BC fit nulla, et reducendo has æquationes, ponendo $BC = 0$, habentur valores dissipationum successivarum, pro quovis lentium numero:

$$1. \overset{I}{B}\overset{I}{C} = B^2 \times \frac{2n}{\overset{I}{N} - \overset{I}{I} \cdot \overset{I}{P}}.$$

$$2. \overset{II}{B}\overset{II}{C} = \overset{I}{B}^2 \times \frac{2n}{\overset{II}{N} - \overset{II}{I} \cdot \overset{II}{P}} + \frac{\overset{I}{B}^2 \cdot B^2}{A^2} \times \frac{2n}{\overset{I}{N} - \overset{I}{I} \cdot \overset{I}{P}}.$$

$$3. \overset{III}{B}\overset{III}{C} = \overset{II}{B}^2 \times \frac{2n}{\overset{III}{N} - \overset{III}{I} \cdot \overset{III}{P}} + \frac{\overset{II}{B}^2 \cdot \overset{I}{B}^2}{A^2} \times \frac{2n}{\overset{I}{N} - \overset{I}{I} \cdot \overset{I}{P}} + \frac{\overset{II}{B}^2 \cdot \overset{I}{B}^2 \cdot B^2}{A^2 \cdot A^2} \times \frac{2n}{\overset{I}{N} - \overset{I}{I} \cdot \overset{I}{P}}.$$

$$4. \overset{IV}{B}\overset{IV}{C} = \overset{III}{B}^2 \times \frac{2n}{\overset{IV}{N} - \overset{IV}{I} \cdot \overset{IV}{P}} + \frac{\overset{III}{B}^2 \cdot \overset{II}{B}^2}{A^2} \times \frac{2n}{\overset{II}{N} - \overset{II}{I} \cdot \overset{II}{P}} + \frac{\overset{III}{B}^2 \cdot \overset{II}{B}^2 \cdot \overset{I}{B}^2}{A^2 \cdot A^2} \times \frac{2n}{\overset{I}{N} - \overset{I}{I} \cdot \overset{I}{P}} + \frac{\overset{III}{B}^2 \cdot \overset{II}{B}^2 \cdot \overset{I}{B}^2 \cdot B^2}{A^2 \cdot A^2 \cdot A^2 \cdot A^2} \times \frac{2n}{\overset{I}{N} - \overset{I}{I} \cdot \overset{I}{P}}.$$

Et ita deinceps, si plures fuerint lentes.

§ 18.

Datis positionibus lentium quocunque in communi axe, una cum singularum legibus refractionum pro radiis omnis generis, oporteat invenire relationem inter distantias focorum principalium earundem lentium, quæ efficiat, ut radii heterogenei à quovis puncto advenientes vel paralleli, post refractionem in omnibus lentibus emergant, sine dissipatione à diversa refrangibilitate radiorum oriunda, si fieri potest.

Solutio problematis generaliter absolvitur ponendo expressionem dissipationis, datæ multitudini lentium convenientem, § 17, nihilo æqualem, quo ipso habetur æquatio definiens relationum distantiarum focalium, $P, \overset{I}{P}, \overset{II}{P}, \overset{III}{P}, \&c.$ requisitam ad id, ut dissipatio in foco ultimo evanescat. Ita in casu binarum

$$\text{lentium habebitur: } \frac{\overset{I}{n}}{\overset{I}{N} - \overset{I}{1} \cdot \overset{I}{P}} + \frac{B^2}{\overset{I}{A}^2} \times \frac{\overset{II}{n}}{\overset{II}{N} - \overset{II}{1} \cdot \overset{II}{P}} = 0;$$

$$\text{in casu trium lentium erit: } \frac{\overset{II}{n}}{\overset{II}{N} - \overset{II}{1} \cdot \overset{II}{P}} + \frac{\overset{I}{B}^2}{\overset{II}{A}^2} \times \frac{\overset{I}{n}}{\overset{I}{N} - \overset{I}{1} \cdot \overset{I}{P}}$$

$$+ \frac{\overset{I}{B}^2 \cdot \overset{II}{B}^2}{\overset{II}{A}^2 \cdot \overset{I}{A}^2} \times \frac{\overset{III}{n}}{\overset{III}{N} - \overset{III}{1} \cdot \overset{III}{P}} = 0; \text{ in casu quatuor lentium:}$$

$$\frac{\overset{III}{n}}{\overset{III}{N} - \overset{III}{1} \cdot \overset{III}{P}} + \frac{\overset{II}{B}^2}{\overset{III}{A}^2} \times \frac{\overset{II}{n}}{\overset{II}{N} - \overset{II}{1} \cdot \overset{II}{P}} + \frac{\overset{II}{B}^2 \cdot \overset{I}{B}^2}{\overset{III}{A}^2 \cdot \overset{II}{A}^2} \times \frac{\overset{I}{n}}{\overset{I}{N} - \overset{I}{1} \cdot \overset{I}{P}}$$

$$+ \frac{\overset{II}{B}^2 \cdot \overset{III}{B}^2 \cdot \overset{I}{B}^2}{\overset{III}{A}^2 \cdot \overset{II}{A}^2 \cdot \overset{I}{A}^2} \times \frac{\overset{IV}{n}}{\overset{IV}{N} - \overset{IV}{1} \cdot \overset{IV}{P}} = 0; \text{ et ita porro, si plures}$$

fuerint lentes.

Ita

Ita si in æquatione pro binis lentibus, $\frac{\frac{1}{n}}{N - 1 \cdot P}$
 $+ \frac{B^2}{A^2} \times \frac{n}{N - 1 \cdot P} = 0$, ponatur distantia lentium
 $B + A = 0$, ut evadant contiguæ, quemadmodum
 in vitro objectivo Dollondiano; habetur $\frac{\frac{1}{n}}{N - 1 \cdot P}$
 $+ \frac{n}{N - 1 \cdot P} = 0$. Unde patet alterutram lentium P
 et P faciendam esse concavam, alteram convexam,
 et distantias earum focales affirmative sumtas P et P
 faciendas esse in ratione $\frac{n}{N - 1}$ ad $\frac{\frac{1}{n}}{N - 1}$, quæ ratio
 evadit n ad $\frac{1}{n}$, si vires refractivæ mediæ utriusque
 lentis fuerint æquales.

Similiter fit construendum telescopium ex tribus
 lentibus, quarum duæ priores P et P juxta se positæ
 constituent vitrum objectivum, tertia P sit ocularis,
 et potentia amplificandi exprimitur numero m .
 Quoniam lentes priores P et P sunt juxta se positæ,
 erit earum distantia $B + A = 0$. Quoniam per con-
 stitutionem telescopii radii in primam lentem inci-
 dentes sunt paralleli, erit $B = P$, et $\frac{1}{B} = \frac{1}{P} + \frac{1}{P}$.
 Quoniam radii ex ultima lente emergunt paralleli,
 erit $A = P$. Et tandem, quia potentia amplificandi
 5 indicatur

indicatur numero m , erit $B = m P$. Substitutis itaque his valoribus in æquatione pro tribus lentibus

supra allata, tranfit illa in hanc: $\frac{n}{N-1} \cdot P + \frac{n}{N-1} \cdot P$

$+ \frac{n}{N-1 \cdot m^2 P} = 0$. Et hæc æquatio una cum æqua-

tione $\frac{1}{P} + \frac{1}{P} - \frac{1}{m P} = 0$, determinat rectationem

distantiarum focalium P , P , P , qualis ad id requiritur, ut radiorum emergentium dissipatio à diversa refrangibilitate nulla fit. Comparando scilicet has

æquationes, habetur $\left(\frac{nm}{N-1} + \frac{n}{N-1} \right) P + \left(\frac{nm}{N-1} \right.$

$\left. \frac{n}{N-1} \right) P = 0$, et $\left(\frac{n}{N-1} - \frac{n}{N-1} \right) \times m^2 P$

$+ \left(\frac{nm}{N-1} + \frac{n}{N-1} \right) \times P = 0$. Eadem ratione præ-

cedendum est in aliis casibus.

S. Klingenstierna.

Stockholm,
20 Aprilis 1760.